Welfare for Economy under Awareness (Algebra, Languages and Computation)

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Welfare for Economy under Awareness

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Abstract. We present the extended notion of pure exchange economy under uncertainty, called an *economy with awareness structure*, where each trader having a strictly monotone preference makes decision under his/her awareness and belief, and we introduce a generalized notion of equilibrium for the economy, called an *expectations equilibrium in awareness*. We show the existence theorem of the equilibrium and the fundamental welfare theorem for the economy, i.e., an allocation in the economy is ex-ante Pareto optimal if and only if it is an expectations equilibrium allocation in awareness.

1 Introduction

This article relates economies and distributed belief. We shall present a generalized notion of economy under uncertainty, called an *economy with awareness structure*, where each trader makes decision in his/her awareness and belief under incomplete information. The purposes are: First, to introduce an extended notion of expectations equilibrium for the economy, called an *expectations equilibrium in awareness*. Secondly, to show the fundamental welfare theorem for the extended economy under expectations equilibrium in awareness.

Main Theorem. *In a pure exchange economy under uncertainty, the traders are assumed to have an awareness structure and they are risk averse. Then an allocation in the economy is ex-ante Pareto optimal if and only if it is an expectations equilibrium allocation in awareness for some initial endowment with respect to some price system.*

In Economic theory and its related fields, many authors have investigated several notions of equilibrium in an economy under asymmetric information.[1] They have studied the relationships between these equilibrium concepts (e.g.: The existence theorem of equilibrium, the core equivalence theorem and the no trade theorem etc.) One of the serious limitations of their analysis is to assume

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[1] See the literatures cited in F. Forges, E. Minelli, and R. Vohla, *Incentive and the core of exchange economy - Survey*, Journal of Mathematical Economics 38 (2002), 1–41.

'partition' structure as information the traders receive. From the epistemic point of view, the partition structure represents the traders' knowledge satisfying the postulates: 'Truth' **T** (what is known is true), the 'positive introspection' **4** (that we know what we do) and the 'negative introspection' **5** (that we know what we do not know). The postulate **5** is indeed so strong that describes the hyper-rationality of traders, and thus it is particularly objectionable.

This raises the question to what extent results on the information partition structure (or the equivalent postulates of knowledge). The answer is to strengthen the results: We shall weaken the conditions in the partition. This relaxation can potentially yield important results in a world with imperfectly Bayesian agents.

The idea has been performed in different settings. Geanakoplos [5] showed the no speculation theorem in the extended rational expectations equilibrium under the assumption that the information structure is reflexive, transitive and *nested.* The condition 'nestedness' is interpreted as a requisite on the 'memory' of the trader. Einy et al [4] extended the core equivalence theorem of Aumann [1] to the equivalence theorem between the ex-post core and the rational expectations equilibria for an economy under asymmetric information. Recently, Matsuhisa [6] gives an extension of the theorem into an economy under awareness structure. In his line we establish the fundamental theorem for welfare into the generalized economy.

This article is organized as follows: In Section 2 we propose the model: An economy with awareness structure and an expectations equilibrium in awareness. In Section 3 we state explicitly the fundamental theorem for welfare economics and sketch the proof. Finally we conclude by remarks.

2 The Model

Let Ω be a non-empty *finite* set called a *state space* and 2^{Ω} the field 2^{Ω} consisting of all subsets of Ω. Each member of 2^{Ω} is called an *event* and each element of Ω called a *state*. We denote by T the set of the *traders*. We shall present a model of awareness according to E. Dekel et al [3].[2]

2.1 Awareness, Belief and Information

A *belief structure* is a tuple $\langle \Omega, (B_t)_{t\in T}\rangle$ in which $B_t : 2^{\Omega} \to 2^{\Omega}$ is trader t's *belief* operator. The interpretation of the event B_tE is that 't believes E.' An *awareness structure* is a tuple $\langle \Omega, (A_t)_{t\in T}, (B_t)_{t\in T}\rangle$ in which $\langle \Omega, (B_t)_{t\in T}\rangle$ is a belief structure and A_t is t's *awareness* operator on 2^{Ω} defined by

$$\textbf{PL} \qquad A_tE = B_tE \cup B_t(\Omega \setminus B_tE) \quad \text{for every } E \text{ in } 2^{\Omega}.$$

[2] A different approach of awareness models is discussed in R. Fagin, J.Y. Halpern, Y. Moses and M.Y. Vardi, *Reasoning about Knowledge.* The MIT Press, Cambridge, Massachusetts, London, England, 1995.

The interpretation of $A_t E$ is that 't is aware of E.' The property **PL** says that t is aware of E if he believes it or if he believes that he does not believe it.

We shall give the generalized notion of information partition in the line of Bacharach [2] as follows.

Definition 1. The *associated information structure* $(P_t)_{t\in T}$ with awareness structure $\langle \Omega, (A_t), (B_t)\rangle$ is the class of t's *associated information functions* $P_t : \Omega \to 2^\Omega$ defined by $P_t(\omega) = \bigcap_{E\in 2^\Omega}\{E \mid \omega \in B_t E\}$. (If there is no event E for which $\omega \in B_t E$ then we take $P_t(\omega)$ to be undefined.) We denote by $\mathrm{Dom}(P_t)$ the set $\{\omega \in \Omega \mid P_t(\omega) \neq \emptyset\}$, called the *domain of* P_t.

The mapping P_t is called *reflexive* if:

Ref $\quad \omega \in P_t(\omega) \quad$ for every $\omega \in \mathrm{Dom}(P_t)$,

and it is said to be *transitive* if:

Trn $\quad \xi \in P_t(\omega)$ implies $P_t(\xi) \subseteq P_t(\omega)$ for any $\xi, \omega \in \mathrm{Dom}(P_t)$.

Furthermore P_t is called *symmetric* if:

Sym $\quad \xi \in P_t(\omega)$ implies $P_t(\xi) \ni \omega$ for any ω and $\xi \in \mathrm{Dom}(P_t)$.

Remark 1. M. Bacharach [2] introduces the *strong epistemic model* equivalent to the Kripke semantics of the modal logic **S5**. The strong epistemic model is a tuple $\langle \Omega, (K_t)_{t\in T}\rangle$ in which t's *knowledge operator* $K_t : 2^\Omega \to 2^\Omega$ satisfies the five postulates: For every E, F of 2^Ω,

N $K_t\Omega = \Omega$,	**K** $K_t(E \cap F) = K_t E \cap K_t F$,	**T** $K_t F \subseteq F$;
4 $K_t F \subseteq K_t K_t F$,	**5** $\Omega \setminus K_t F \subseteq K_t(\Omega \setminus K_t F)$.	

t's associated information function P_t induced by K_t makes a partition of Ω, called t's *information partition*, which satisfies the postulates **Ref**, **Trn** and **Sym**. This is just the Kripke semantics corresponding to the logic **S5**; the postulates **Ref**, **Trn** and **Sym** are respectively equivalent to the postulates **T**, **4** and **5**. The strong epistemic model can be interpreted as the awareness structure $\langle \Omega, (A_t), (B_t)\rangle$ such that B_t is the knowledge operator. In this situation it is easily verified that A_t must be the *trivial* operator,[3] and that $\mathrm{Dom}(P_t) = \Omega$.

2.2 Economy with Awareness Structure

A pure exchange economy *under uncertainty* is a structure

$$\mathcal{E} = \langle T, \Omega, \mathbf{e}, (U_t)_{t\in T}, (\pi_t)_{t\in T}\rangle$$

consisting of the following structure and interpretations: There are l commodities in each state of the state space Ω; the consumption set of each trader t is $\mathbf{R}^l_+$; an *initial endowment* is a mapping $\mathbf{e} : T \times \Omega \to \mathbf{R}^l_+$ with which $\mathbf{e}(t,\cdot) : \Omega \to \mathbf{R}^l_+$ is called t's *initial endowment*; $U_t : \mathbf{R}^l_+ \times \Omega \to \mathbf{R}$ is t's von-Neumann and Morgenstern utility function; π_t is a subjective prior on Ω for $t \in T$. For simplicity, π_t is assumed to be *full support* for all $t \in T$. That is, $\pi_t(\omega) \neq 0$ for every $\omega \in \Omega$.

[3] I.e. $A_t(F) = \Omega$ for every $F \in 2^\Omega$

Definition 2. A pure exchange economy *with awareness structure* is a structure $\mathcal{E}^A = \langle \mathcal{E}, (A_t)_{t\in T}, (B_t)_{t\in T}, (P_t)_{t\in T}\rangle$, in which $\mathcal{E}$ is a pure exchange economy under uncertainty, and $\langle \Omega, (A_t)_{t\in T}, (B_t)_{t\in T}, (P_t)_{t\in T}\rangle$ is an awareness structure with $(P_t)_{t\in T}$ the associated information structure. By the *domain* of the economy $\mathcal{E}^A$ we mean $\mathrm{Dom}(\mathcal{E}^A) = \cap_{t\in T}\mathrm{Dom}(P_t)$. We always assume that $\mathrm{Dom}(\mathcal{E}^A) \neq \emptyset$.

Remark 2. An economy under asymmetric information is an economy $\mathcal{E}^A$ with the awareness structure $\langle \Omega, (A_t)_{t\in T}, (B_t)_{t\in T}\rangle$ given by the strong epistemic model, and that $\mathrm{Dom}(\mathcal{E}^A) = \Omega$.

We denote by $\mathcal{F}_t$ the field of $\mathrm{Dom}(P_t)$ generated by $\{P_t(\omega) | \ \omega \in \Omega\}$ and denote by $\Pi_t(\omega)$ the atom containing $\omega \in \mathrm{Dom}(P_t)$. We denote by $\mathcal{F}$ the join of all $\mathcal{F}_t (t \in T)$ on $\mathrm{Dom}(\mathcal{E}^A)$; i.e. $\mathcal{F} = \vee_{t\in T}\mathcal{F}_t$, and denote by $\{\Pi(\omega) | \ \omega \in \mathrm{Dom}(\mathcal{E}^A)\}$ the set of all atoms $\Pi(\omega)$ containing ω of the field $\mathcal{F} = \vee_{t\in T}\mathcal{F}_t$. We shall often refer to the following conditions: For every $t \in T$,

A-1 $\sum_{t\in T} \mathbf{e}(t,\omega) > 0$ for each $\omega \in \Omega$.
A-2 $\mathbf{e}(t,\cdot)$ is $\mathcal{F}$-measurable on $\mathrm{Dom}(P_t)$.
A-3 For each $x \in \mathbf{R}^l_+$, the function $U_t(x,\cdot)$ is at least $\mathcal{F}$-measurable on $\mathrm{Dom}(\mathcal{E}^A)$, and the function: $T \times \mathbf{R}^l_+ \to \mathbf{R}$, $(t,x) \mapsto U_t(x,\omega)$ is $2^\Omega \times \mathcal{B}$-measurable where $\mathcal{B}$ is the σ-field of all Borel subsets of $\mathbf{R}^l_+$.
A-4 For each $\omega \in \Omega$, the function $U_t(\cdot,\omega)$ is strictly increasing on $\mathbf{R}^l_+$, continuous, strictly quasi-concave and *non-satiated* on $\mathbf{R}^l_+$.[4]

2.3 Expectations Equilibrium in Awareness

An *assignment* is a mapping $\mathbf{x} : T \times \Omega \to \mathbf{R}^l_+$ such that for each $t \in T$, the function $\mathbf{x}(t,\cdot)$ is at least $\mathcal{F}$-measurable on $\mathrm{Dom}(\mathcal{E}^A)$. We denote by $\mathcal{A}ss(\mathcal{E}^A)$ the set of all assignments for the economy $\mathcal{E}^A$. By an *allocation* we mean an assignment $\mathbf{a}$ such that $\mathbf{a}(t,\cdot)$ is $\mathcal{F}$-measurable on $\mathrm{Dom}(\mathcal{E}^A)$ for all $t \in T$ and $\sum_{t\in T}\mathbf{a}(t,\omega) \leq \sum_{t\in T}\mathbf{e}(t,\omega)$ for every $\omega \in \Omega$. We denote by $\mathcal{A}lc(\mathcal{E}^A)$ the set of all allocations.

We introduce the revised notion of trader's expectation of utility in $\mathcal{E}^A$. By t's *ex-ante* expectation we mean $\mathbf{E}_t[U_t(\mathbf{x}(t,\cdot)] := \sum_{\omega\in\mathrm{Dom}(P_t)} U_t(\mathbf{x}(t,\omega),\omega)\pi_t(\omega)$ for each $\mathbf{x} \in \mathcal{A}ss(\mathcal{E}^A)$. The *interim* expectation $\mathbf{E}_t[U_t(\mathbf{x}(t,\cdot)|P_t]$ is defined by $\mathbf{E}_t[U_t(\mathbf{x}(t,\cdot)|P_t](\omega) := \sum_{\xi\in\mathrm{Dom}(P_t)} U_t(\mathbf{x}(t,\xi),\xi)\pi_t(\{\xi\} \cap A_t(\{\xi\})|P_t(\omega))$ on $\mathrm{Dom}(P_t)$. It should be noted that we use not the usual notion of posterior $\pi_t(\{\xi\}|P_t(\omega))$ but the revised one $\pi_t(\{\xi\} \cap A_t(\{\xi\})|P_t(\omega))$.[5]

A *price system* is a non-zero function $p : \Omega \to \mathbf{R}^l_+$ which is $\mathcal{F}$-measurable on $\mathrm{Dom}(\mathcal{E}^A)$. We denote by $\Delta(p)$ the partition on Ω induced by p, and denote by $\sigma(p)$ the field of Ω generated by $\Delta(p)$. The *budget set* of a trader t at a state ω for a price system p is defined by $B_t(\omega,p) := \{ \ x \in \mathbf{R}^l_+ \ | \ p(\omega)\cdot x \leq p(\omega)\cdot$

[4] That is, for any $x \in \mathbf{R}^l_+$ there exists an $x' \in \mathbf{R}^l_+$ such that $U_i(x',\omega) > U_i(x,\omega)$.
[5] A discussion why this improvement of the notion of posterior is needed is given in T. Matsuhisa and S.-S. Usami, *Awareness, belief and agreeing to disagree*, Far East Journal of Mathematical Sciences 2(6) (2000) 833–844.

$\mathbf{e}(t,\omega)$ }. Define the mapping $\Delta(p) \cap P_t : \mathrm{Dom}(P_t) \to 2^{\Omega}$ by $(\Delta(p) \cap P_t)(\omega) := \Delta(p)(\omega) \cap P_t(\omega)$. We denote by $\mathrm{Dom}(\Delta(p) \cap P_t)$ the set of all states ω in which $\Delta(p)(\omega) \cap P_t(\omega) \neq \emptyset$. Let $\sigma(p) \vee \mathcal{F}_t$ be the smallest σ-field containing both the fields $\sigma(p)$ and $\mathcal{F}_t$.

Definition 3. An *expectations equilibrium in awareness* for an economy $\mathcal{E}^A$ with awareness structure is a pair $(p, \mathbf{x})$, in which p is a price system and $\mathbf{x}$ is an assignment satisfying the following conditions:

EA1 $\mathbf{x}$ is an allocation;
EA2 For all $t \in T$ and for every $\omega \in \Omega$, $\mathbf{x}(t,\omega) \in B_t(\omega, p)$;
EA3 For all $t \in T$, if $\mathbf{y}(t,\cdot) : \Omega \to \mathbf{R}^l_+$ is $\mathcal{F}$-measurable on $\mathrm{Dom}(\mathcal{E}^A)$ with $\mathbf{y}(t,\omega) \in B_t(\omega, p)$ for all $\omega \in \Omega$, then

$$\mathbf{E}_t[U_t(\mathbf{x}(t,\cdot))|\Delta(p) \cap P_t](\omega) \geq \mathbf{E}_t[U_t(\mathbf{y}(t,\cdot))|\Delta(p) \cap P_t](\omega)$$

pointwise on $\mathrm{Dom}(\Delta(p) \cap P_t)$;
EA4 For every $\omega \in \mathrm{Dom}(\mathcal{E}^A)$, $\sum_{t\in T} \mathbf{x}(t,\omega) = \sum_{t \in T} \mathbf{e}(t,\omega)$.

The allocation $\mathbf{x}$ in $\mathcal{E}^A$ is called an expectations equilibrium *allocation* in awareness for $\mathcal{E}^A$.

We denote by $EA(\mathcal{E}^A)$ the set of all the expectations equilibria of a pure exchange economy $\mathcal{E}^A$, and denote by $\mathcal{A}(\mathcal{E}^A)$ the set of all the expectations equilibrium allocations in awareness for the economy.

3 The Result

Let $\mathcal{E}^A$ be the economy with awareness structure and $\mathcal{E}^A(\omega)$ the economy with complete information $\langle T, (\mathbf{e}(t,\omega))_{t\in T}, (U_t(\cdot,\omega))_{t\in T}\rangle$ for each $\omega \in \Omega$. We denote by $\mathcal{W}(\mathcal{E}^A(\omega))$ the set of all competitive equilibria for $\mathcal{E}^A(\omega)$.

An allocation $\mathbf{x}$ in $\mathcal{E}^A$ is said to be *ex-ante Pareto-optimal* if there is no allocation $\mathbf{a}$ such that $\mathbf{E}_t[U_t(\mathbf{a}(t,\cdot))] \geq \mathbf{E}_t[U_t(\mathbf{x}(t,\cdot))]$ for all $t \in T$ with at least one inequality strict. We can now state our main theorem.

Theorem 1. *Let $\mathcal{E}^A$ be an economy with awareness structure satisfying the conditions* **A-1**, **A-2**, **A-3** *and* **A-4**. *An allocation is ex-ante Pareto optimal if and only if it is an expectations equilibrium allocation in awareness for some initial endowment* $\mathbf{w}$ *with respect to some price system such that* $\sum_{t\in T} \mathbf{w}(t,\omega) = \sum_{t\in T} \mathbf{e}(t,\omega)$ *for each* $\omega \in \mathrm{Dom}(\mathcal{E}^A)$.

Proof. Follows immediately from Propositions 1 and 2 as below.

Proposition 1. *Let $\mathcal{E}^A$ be an economy with awareness structure satisfying the conditions* **A-1**, **A-2**, **A-3** *and* **A-4**. *Then an allocation* $\mathbf{x}$ *is ex-ante Pareto optimal if it is an expectations equilibrium allocation in awareness with respect to some price system.* □

Proposition 2. *Let $\mathcal{E}^A$ be an economy with awareness structure satisfying the conditions* **A-1**, **A-2**, **A-3** *and* **A-4**. *If an allocation* $\mathbf{x}$ *is ex-ante Pareto optimal in* $\mathcal{E}^A$ *then there are a price system and an initial endowment* $\mathbf{e}'$ *such that* $\mathbf{x}$ *is an expectations equilibrium allocation in awareness with* $\sum_{t \in T} \mathbf{e}'(t, \omega) = \sum_{t \in T} \mathbf{e}(t, \omega)$ *for each* $\omega \in \mathrm{Dom}(\mathcal{E}^A)$. □

4 Concluding Remarks

Our real concern in this article is about relationship between players' beliefs and their decision making, especially when and how the players take corporate actions under their decisions. We focus on extending the fundamental theorem of welfare economics into an economy with traders having 'awareness and belief' model. We have shown that the nature of the theorem is dependent not on common-belief nor on the partition structure of traders' information, but on the structure of awareness and belief when each player receives information.

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